

Rozmyte sieci kognitywne w modelowaniu i predykcji aproksymowanych szeregów czasowych

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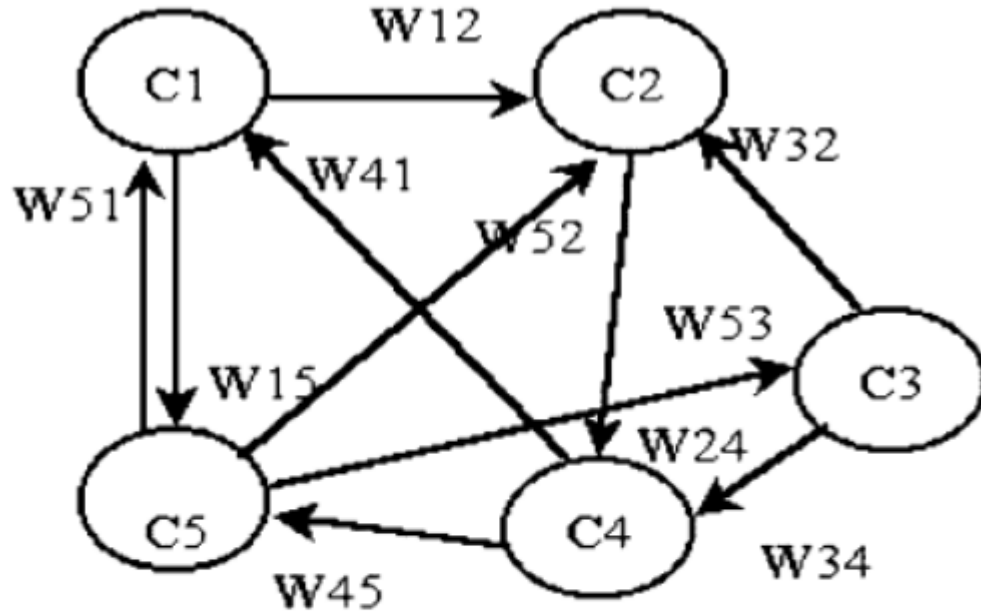
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Plan referatu

- Wprowadzenie do sieci FCM
- Przedziałowe szeregi czasowe – predykcja za pomocą szarych sieci FCM
- Granularne szeregi czasowe - dwuetapowa granularyzacja oraz predykcja z zastosowaniem FCM

Fuzzy Cognitive Maps



$$\text{FCM} = \langle C, W \rangle$$

$$c(t) \in [0, 1]$$

$$t \in [0, 1, \dots, t_e], t_e \in \mathbb{N}$$

$$c'_j(t) = f\left(\sum_{i=1, i \neq j}^n w_{ij} c_i(t-1)\right)$$

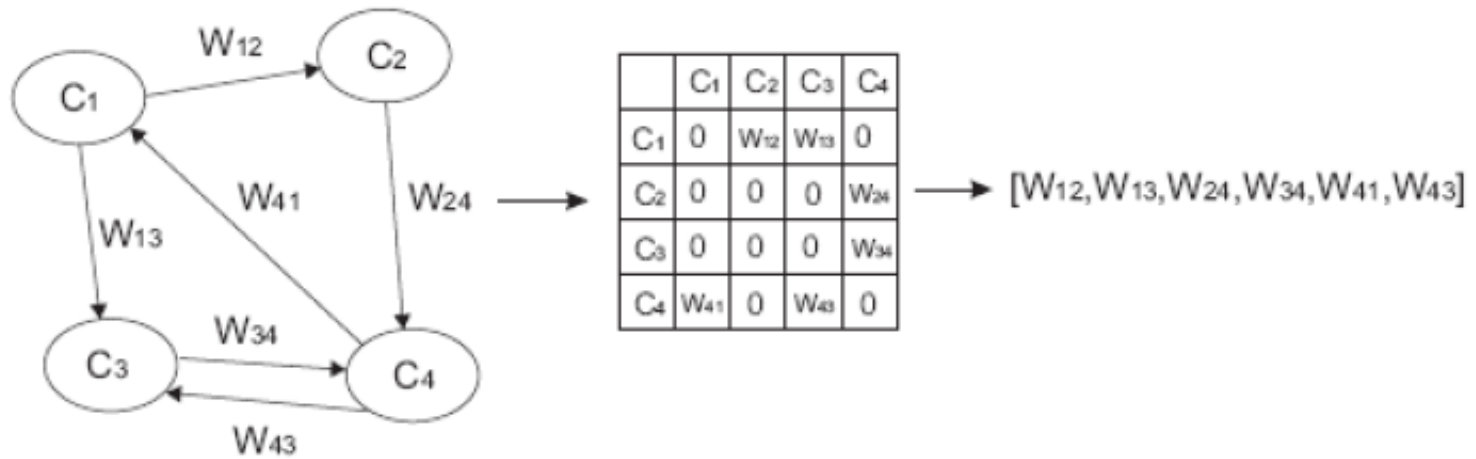
$$f(x) = \frac{1}{1 + e^{-\lambda x}}$$

$$w_{ij} \in [-1, 1]$$

W =

	C1	C2	C3	C4	C5
C1	0.5	0	1	0	0
C2	0.25	0	-0.5	0	0
C3	0	0.5	0	-0.5	-0.5
C4	0.25	0.5	0	0	0
C5	0.5	0	0	0.5	0

Uczenie FCM za pomocą algorytmów populacyjnych



$$fitness = \frac{1}{\epsilon + 1}$$

$$\epsilon = \frac{1}{(t_e - 1) \cdot n} \cdot \sum_{t=2}^{t_e} \sum_{i=1}^n |c_i(t) - c'_i(t)|$$



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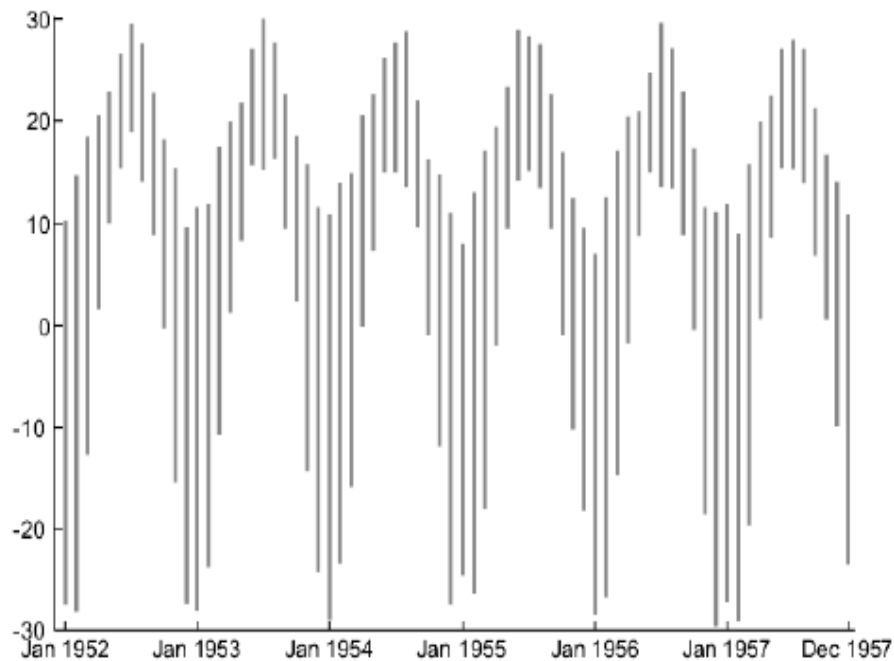
Evolutionary learning of fuzzy grey cognitive maps for the forecasting of multivariate, interval-valued time series [☆]

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Przedziałowe szeregi czasowe



$$v \in V \quad v(\tau) \quad \tau \in \mathfrak{R}, \tau \in [0, \tau_e]$$

$$\{V(\tau)\}$$

$$[0, 1], (1, 2), \dots, (t_e - 1, t_e]$$

$$\tau \in (t - 1, t]$$

$$\underline{g}(t) = \min(v(\tau))$$

$$\bar{g}(t) = \max(v(\tau)), \tau \in (t - 1, t]$$

$$\otimes g_i(t) = [\underline{g}_i(t), \bar{g}_i(t)] \quad t \in [1, t_e]$$

$$\otimes g_i \in \otimes G$$

$$\{\otimes G(t)\} = \{\otimes G(1), \otimes G(2), \dots, \otimes G(n)\}$$

$$ITS = \{\otimes G(t)\}$$

Błędy predykcji dla przedziałowych szeregów czasowych

$$MDE_{IY} = \frac{1}{t_e - 1} \cdot \sum_{t=2}^{t_e} 0.5 \cdot (|\underline{g} - \underline{g}'| + |\bar{g} - \bar{g}'|)$$

Ichino-Yaguchi

$$MDE_H = \frac{1}{t_e - 1} \cdot \sum_{t=2}^{t_e} (|g_c - g'_c| + |g_r - g'_r|)$$

$g_c = \frac{g + \bar{g}}{2}$
 $g_r = \frac{\bar{g} - g}{2}$

$$MDE_K = \frac{1}{t_e - 1} \cdot \sum_{t=2}^{t_e} \frac{1}{\sqrt{2}} \cdot \sqrt{(\underline{g}(t) - \underline{g}'(t))^2 + (\bar{g}(t) - \bar{g}'(t))^2}$$

Dotychczasowe prace dotyczące predykcji szeregów przedziałowych

Type	Model		
	Lower and upper bounds seperately	Lower and upper bounds combined	Interval arithmetic
univariate	[10, 11, 12]	[10, 11, 13, 9]	[11, 14, 15]
multivariate	-	-	this paper

Arytmetyka przedziałowa

$$\otimes g \in [\underline{g}, \bar{g}]$$

$$\otimes g_a + \otimes g_b = [\underline{g}_a + \underline{g}_b, \bar{g}_a + \bar{g}_b],$$

$$\otimes g_a \cdot \otimes g_b = \left[\min(\underline{g}_a \cdot \underline{g}_b, \underline{g}_a \cdot \bar{g}_b, \bar{g}_a \cdot \underline{g}_b, \bar{g}_a \cdot \bar{g}_b), \right. \\ \left. \max(\underline{g}_a \cdot \underline{g}_b, \underline{g}_a \cdot \bar{g}_b, \bar{g}_a \cdot \underline{g}_b, \bar{g}_a \cdot \bar{g}_b) \right],$$

$$r \cdot \otimes g = [r \cdot \underline{g}, r \cdot \bar{g}],$$

$$\frac{\otimes g_a}{\otimes g_b} = \left[\min\left(\underline{g}_a \cdot \bar{g}_b^{-1}, \underline{g}_a \cdot \underline{g}_b^{-1}, \bar{g}_a \cdot \bar{g}_b^{-1}, \bar{g}_a \cdot \underline{g}_b^{-1}\right), \right. \\ \left. \max\left(\underline{g}_a \cdot \bar{g}_b^{-1}, \underline{g}_a \cdot \underline{g}_b^{-1}, \bar{g}_a \cdot \bar{g}_b^{-1}, \bar{g}_a \cdot \underline{g}_b^{-1}\right) \right]$$

$$\dot{g} = \delta \cdot \underline{g} + (1 - \delta) \cdot \bar{g} \mid \delta \in [0, 1]$$

Szare, rozmyte sieci kognitywne (FGCM)

$$\underline{c}_i(t) = \frac{g(t) - \underline{g}_{min}}{\bar{g}_{max} - \underline{g}_{min}}, \quad \bar{c}_i(t) = \frac{\bar{g}(t) - \underline{g}_{min}}{\bar{g}_{max} - \underline{g}_{min}}, \quad \underline{g}_{min} = \min(g(t))$$
$$\bar{g}_{max} = \max(\bar{g}(t))$$

$$FGCM = \langle \otimes C, \otimes W, f(\cdot), \eta \rangle, \quad f(x) = \frac{1}{1 + e^{-\lambda \cdot x}}, \quad \eta \in [0, 1]$$

$$\otimes w_{ij} \in [\underline{w}_{ij}, \bar{w}_{ij}] \quad \underline{w}_{ij}, \bar{w}_{ij} \in [-1, +1]$$

Predykcja jeden krok wprzód

$$\otimes c_j(t+1) = \left[\left(1 + e^{-\lambda \cdot \sum_{i=1}^n \underline{\otimes w}_{ij} \cdot \otimes c_i(t)} \right)^{-1}, \left(1 + e^{-\lambda \cdot \sum_{i=1}^n \bar{\otimes w}_{ij} \cdot \otimes c_i(t)} \right)^{-1} \right]$$

$$\underline{\otimes w}_{ij} \cdot \otimes c_i(t) = \min(\underline{w}_{ij} \cdot \underline{c}_i(t), \underline{w}_{ij} \cdot \bar{c}_i(t), \bar{w}_{ij} \cdot \underline{c}_i(t), \bar{w}_{ij} \cdot \bar{c}_i(t))$$

$$\bar{\otimes w}_{ij} \cdot \otimes c_i(t) = \max(\underline{w}_{ij} \cdot \underline{c}_i(t), \underline{w}_{ij} \cdot \bar{c}_i(t), \bar{w}_{ij} \cdot \underline{c}_i(t), \bar{w}_{ij} \cdot \bar{c}_i(t))$$

Algorytm ewolucyjny

Chromosom

- 1) część szara $\otimes W$ $\underline{h}_j = rand(-1, 1)$ $\bar{h}_j = rand(\underline{h}_j, 1)$
- 2) część punktowa parametry określające kształt funkcji transformacji

Funkcja przystosowania

$$fitness = \frac{1}{\epsilon + 1}, \quad \epsilon = \frac{1}{n} \cdot \sum_{i=1}^n ERR(\otimes c_i(t), \otimes c'_i(t)), \quad n = \text{card}(\otimes W)$$

Selekcja elitarna

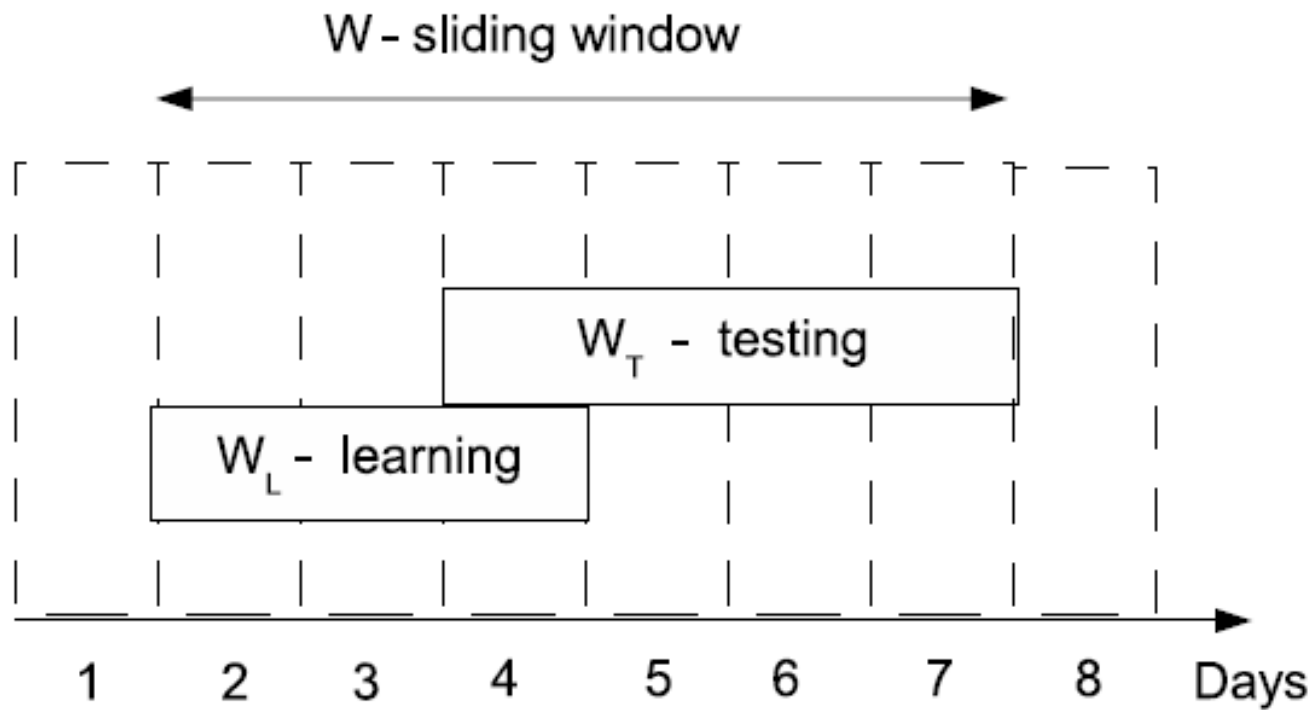
Standardowa mutacja, jednopunktowe krzyżowanie

Eksperymenty - dane meteorologiczne

Table 2: Meteorological variables

v_i	Description
v_1	Dry Bulb [Celsius] The temperature of air measured by a thermometer freely exposed to the air, shielded from radiation and moisture, usually thought of as air temperature.
v_2	Dew Point [Celsius] The temperature below which the water vapor in a volume of humid air at a constant barometric pressure will condense into liquid.
v_3	Relative Humidity
v_4	Wind Speed
v_5	Station Pressure

Eksperymenty



Błędy predykcji

Table 8: Mean MDE_H for out-of-sample errors

L.w. [days]	Testing window [days]			
	2	3	4	5
2	0.2205(0.1193)	0.2301(0.1109)	0.2473(0.0983)	0.2513(0.0952)
3	0.2198(0.1087)	0.2273(0.0951)	0.2442(0.0976)	0.2475(0.0928)
4	0.2179(0.1028)	0.2331(0.0926)	0.2369(0.0822)	0.2363(0.0767)
5	0.2177(0.0995)	0.2273(0.0868)	0.2261(0.0804)	0.2279(0.0697)
6	0.2046(0.0941)	0.2112(0.0787)	0.2144(0.0743)	0.2130(0.0679)
7	0.1971(0.0974)	0.2000(0.0760)	0.2046(0.0706)	0.2041(0.0628)
8	0.1923(0.0922)	0.1977(0.0742)	0.2009(0.0675)	0.2023(0.0617)
9	0.1887(0.0951)	0.1959(0.0752)	0.1984(0.0672)	0.2002(0.0613)
10	0.1898(0.0947)	0.1961(0.0758)	0.1985(0.0660)	0.1995(0.0618)
11	0.1923(0.0957)	0.1937(0.0763)	0.1991(0.0680)	0.2014(0.0630)
12	0.1934(0.0920)	0.1941(0.0749)	0.2004(0.0677)	0.2019(0.0633)
30	0.1922(0.0949)	0.1984(0.0777)	0.2006(0.0695)	0.2015(0.0620)
60	0.1947(0.0957)	0.1988(0.0761)	0.2008(0.0689)	0.2017(0.0620)

Analiza porównawcza

Table 8: Prediction errors for $length(W_L) = 30$ and $length(W_T) = 2$

Method	MDE_{IY}	MDE_H	MDE_K
naive	0.1606(0.0692)	0.2290(0.1087)	0.1798(0.0811)
ARIMA	0.1582(0.0399)	0.2156(0.0526)	0.1705(0.0433)
VAR	0.1423(0.0575)	0.2023(0.0772)	0.1603(0.0643)
ES	0.1407(0.0582)	0.1965(0.0701)	0.1573(0.0627)
FGCM	0.1373(0.0628)	0.1922(0.0949)	0.1528(0.0719)

Table 9: DM test for $length(W_L) = 30$ and $length(W_T) = 2$

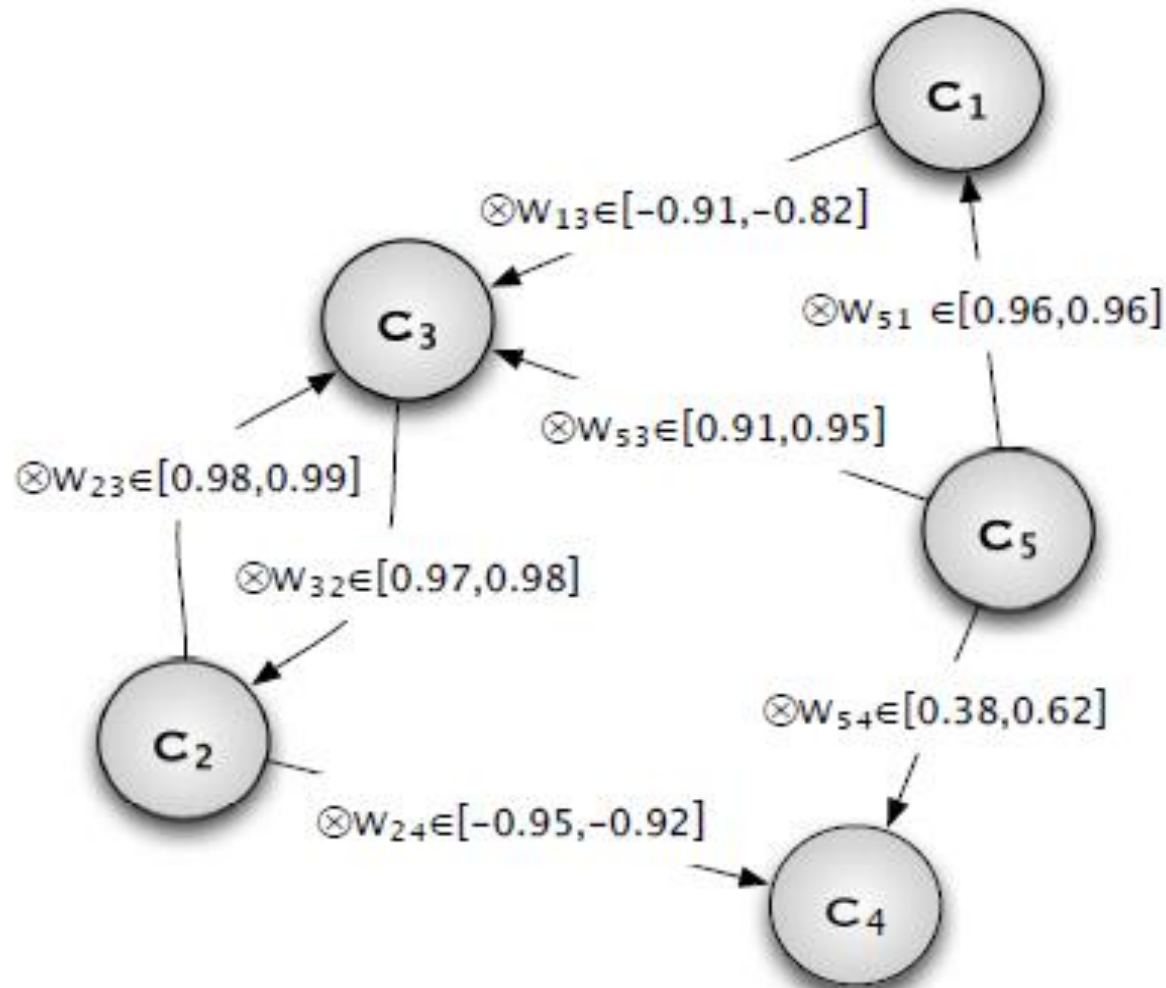
	MDE_{IY}	MDE_H	MDE_K
FGCM/naive	-5.1941(1)	-6.1418(1)	-7.4937(1)
FGCM/ARIMA	-4.7273(1)	-4.625(1)	-6.2510(1)
FGCM/VAR	-1.5536(0.9391)	-1.3057(0.9034)	-2.5106(0.9936)
FGCM/ES	-0.3434(0.6342)	-0.3057(0.6199)	-1.4516(0.9259)

Ograniczenia metody

Table 11: DM test for $length(W_L) = 90$ and $length(W_T) = 4$

	MDE_{IY}	MDE_H	MDE_K
FGCM/naive	-3.1670(0.9991)	-3.2632(0.9994)	-3.5960(0.9998)
FGCM/ARIMA	2.7388(0.0033)	4.4108(0.0008)	7.1973(0.000)
FGCM/VAR	-4.3261(1)	-4.1463(1)	-4.6271(1)
FGCM/ES	-0.2131(0.5192)	-0.3832(0.6481)	-0.2899(0.6139)

Interpretacja graficzna sieci FGCM



Wniosek

Uczenie FGCM za pomocą algorytmu genetycznego wykazuje najlepszą efektywność dla predykcji w krótkim horyzoncie czasowym, przy czym wymagana jest optymalizacja długości okresu, w którym model FGCM jest uczony.

Modelowanie i predykcja granularnych szeregów czasowych



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Fuzzy cognitive maps in the modeling of granular time series

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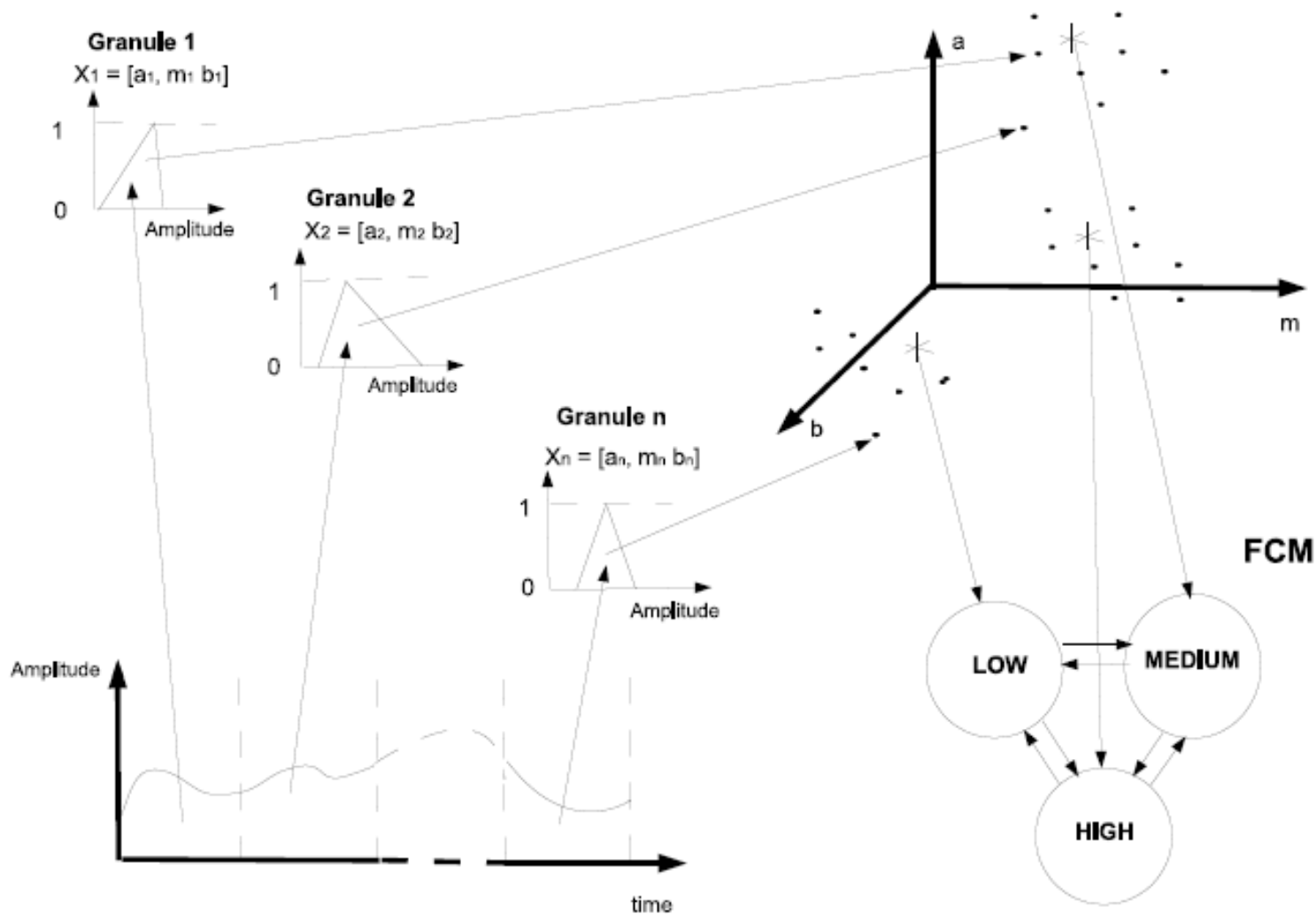
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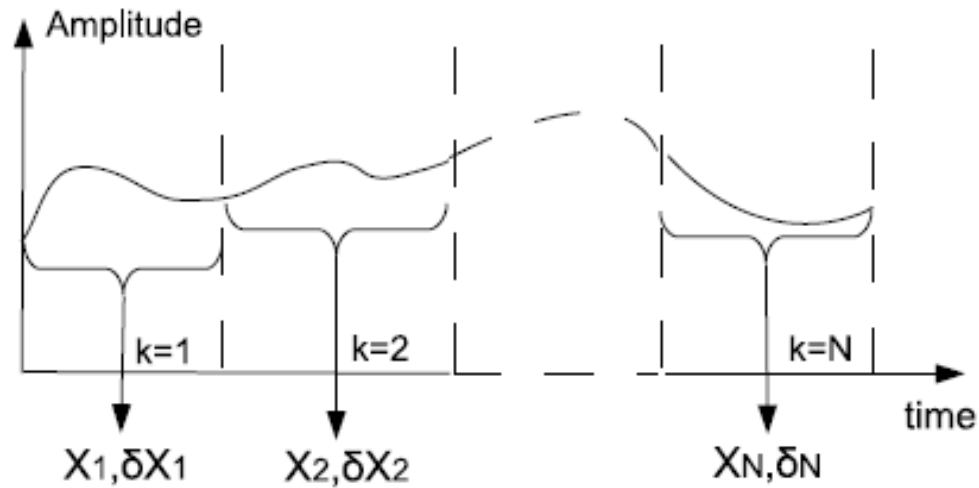
Dwuetapowa aproksymacja szeregu czasowego

First-phase granulation

Second-phase granulation



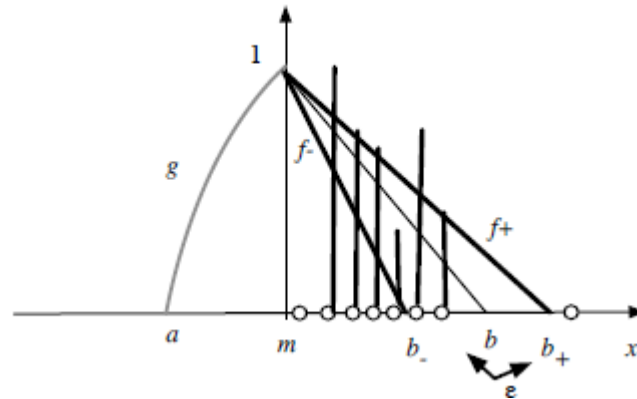
Pierwszy etap granulacji



$$\{X_k\} = \{X_1, X_2, \dots, X_N\} \quad \{\partial X_k\} = \{\partial X_1, \partial X_2, \dots, \partial X_N\}$$

$$\{GX\} = \langle X_1, \partial X_1 \rangle, \langle X_2, \partial X_2 \rangle, \dots, \langle X_N, \partial X_N \rangle$$

Zasada uzasadnionej granularności



$$\mu_{a,m}(x) = \frac{1}{m-a} \cdot x - \frac{a}{m-a}.$$

$$\mu_{m,b}(x) = \frac{1}{m-b} \cdot x - \frac{b}{m-b}$$

$$cov([a, m]) = \sum_{x_k \in [a, m]} \mu(x_k),$$

$$cov([m, b]) = \sum_{x_k \in [m, b]} \mu(x_k).$$

$$sp([a, m]) = 1 - \frac{0.5 \cdot |m - a|}{|x_{max} - m|},$$

$$sp([m, b]) = 1 - \frac{0.5 \cdot |b - m|}{|x_{max} - m|},$$

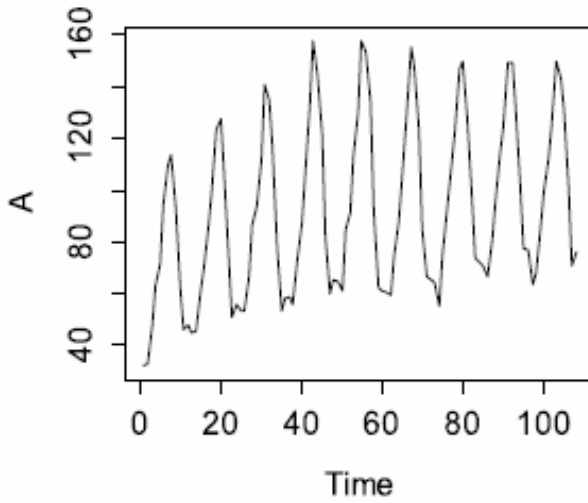
$$Q(a) = cov([a, m]) \cdot sp([a, m]),$$

$$Q(b) = cov([m, b]) \cdot sp([m, b]).$$

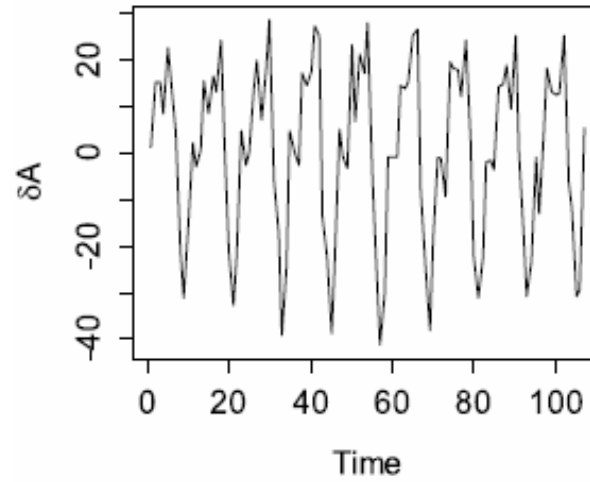
$$a_{opt} = \operatorname{argmax}_a Q(a),$$

$$b_{opt} = \operatorname{argmax}_b Q(b).$$

Przykład



(a) Amplitude



(b) Change of amplitude

Figure 3: Passenger miles (Mil) flown domestic U.K.

Przykład

Table 2: Parameters of the original time series after temporal partitioning

Interval (k)	Time (t)		$x(t)$			$\partial x(t)$		
	from	to	min	m	max	min	m	max
1	1	12	32.2	62.5	113.9	-31.1	4.5	22.6
2	13	24	44.9	71.35	127.4	-33	4.7	24.3
3	25	36	53.2	82.2	140.4	-39.1	4.9	28.7
4	37	48	56.1	85.3	157.8	-39	5	27.1
5	49	60	61.5	92	157.7	-41.3	-1	27.8
6	61	72	59.6	86.55	154.9	-38.3	-1	26.5
7	73	84	55	94.55	149.5	-31	2.7	24.3
8	85	96	66.8	98.3	149.1	-30.9	-0.1	25.1
9	97	108	63.6	99.85	149.6	-30.9	5.5	25.1

Przykład

Table 3: Granular time series $\{GX\}$

Interval (k)	Time (t)		$X(t)$			$\partial X(t)$		
	from	to	a	m	b	a	m	b
1	1	12	36.04	62.5	105.43	-7.63	4.5	22.47
2	13	24	44.9	71.35	118.49	-6.4	4.7	23.19
3	25	36	53.2	82.2	130.48	-29.59	4.9	26.42
4	37	48	56.1	85.3	154.56	-32.78	5	27.09
5	49	60	76.68	92	157.33	-13.16	-1	27.79
6	61	72	59.6	86.55	152.12	-27.54	-1	26.49
7	73	84	55.75	94.55	141.08	-27.2	2.7	24.29
8	85	96	66.8	98.3	142.81	-26.34	-0.1	24
9	97	108	63.6	99.85	147.03	-17.98	5.5	23.86

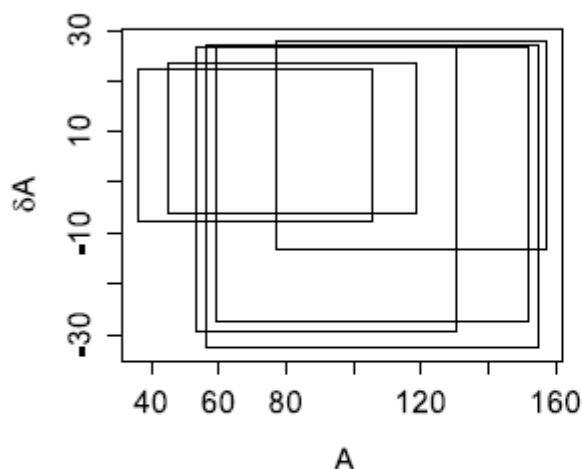
Drugi etap granulacji - grupowanie metodą fuzzy c-means

$$J = \sum_i^N \sum_j^c \mu_{ij}^m \|x_i - a_j\|^2,$$

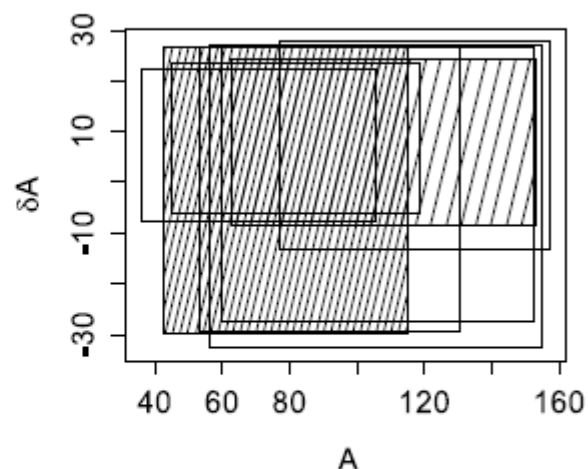
$$\mu_{ij} = \frac{1}{\left(\frac{\|x_i - a_j\|}{\|x_i - a_k\|}\right)^{\frac{2}{m-1}}}. \quad a_j = \frac{\sum_i^N \mu_{ij}^m x_i}{\sum_i^N \mu_{ij}^m}.$$

Warunek stopu: $\max_{ij} (|\mu_{ij}^{k+1} - \mu_{ij}^k|) < \epsilon$

Przykład



(a) Granules



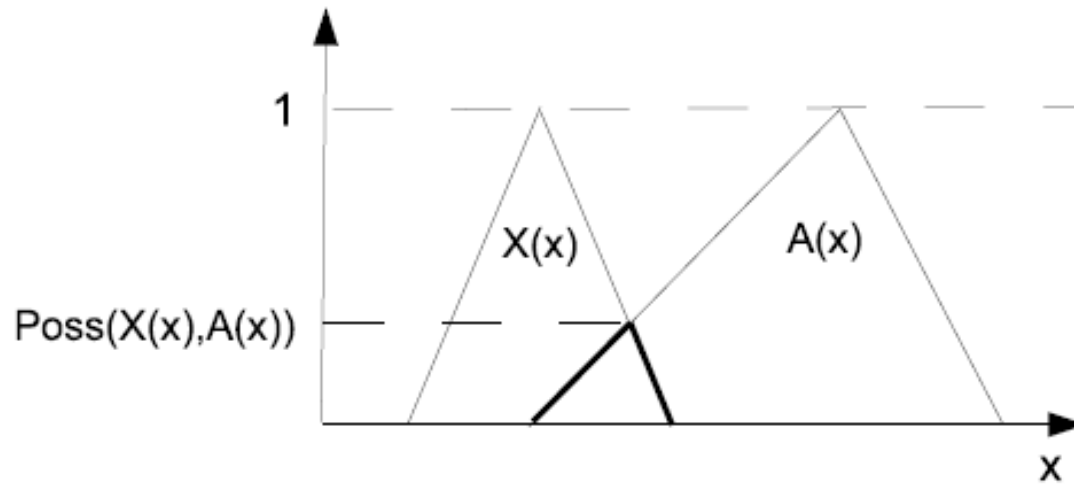
(b) Granules and prototypes

Figure 4: Second stage of granulation

Table 4
Second-phase granules.

j	A_j				∂A_j			
	a	m	b	Term	a	m	b	Term
1	63	87.42	152.83	HIGH	-29.9	3.05	26.68	LOW
2	42.34	69.11	114.67	LOW	-8.73	3.06	24.2	HIGH

Zgodność granul



Alternatywne wersje opisu stosowane jako funkcje Desc i Comp

$$Poss(X_k, A_j) = \max_x \min(X_k(x), A_j(x)), \quad Match(X_k, A_j) = \frac{\sum_x \min(X_k(x), A_j(x))}{\sum_x \max(X_k(x), A_j(x))}.$$
$$Poss(\partial X_k, \partial A_j) = \max_x \min(\partial X_k(x), \partial A_j(x)), \quad Match(\partial X_k, \partial A_j) = \frac{\sum_x \min(\partial X_k(x), \partial A_j(x))}{\sum_x \max(\partial X_k(x), \partial A_j(x))}.$$

Opis granul za pomocą wektorów aktywacji pojęć FCM

$$\lambda_{1j}(k) = Desc(X_k, A_j) \quad \lambda_{2j}(k) = Desc(\partial X_k, \partial A_j)$$

Przykład dla dwóch klastrów

Table 5

Description of {GX} for *Desc = Poss.*

	A_1	A_2	∂A_1	∂A_2
k	$\lambda_{11}(k)$	$\lambda_{12}(k)$	$\lambda_{21}(k)$	$\lambda_{22}(k)$
1	1	0.91	0.96	0.96
2	0.78	0.97	0.95	0.95
3	0.93	0.82	0.97	0.97
4	0.98	0.78	0.97	0.97
5	0.94	1	0.9	0.93
6	0.99	0.76	0.9	0.93

Table 8

Description of {GX} for *Desc = Match.*

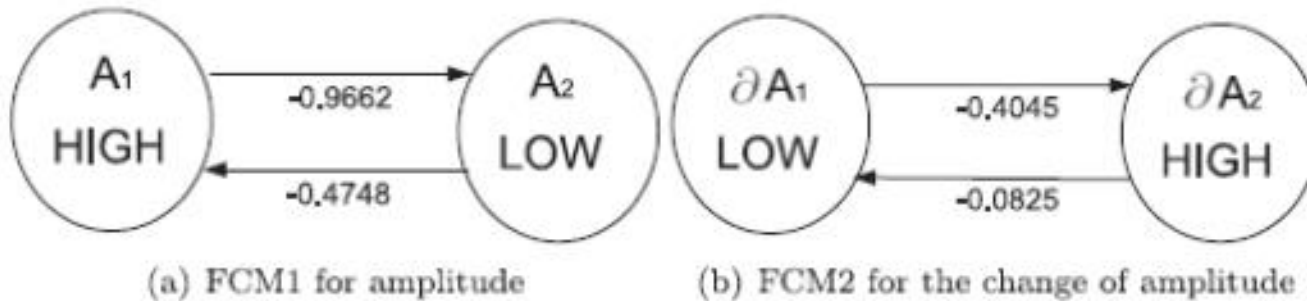
	A_1	A_2	∂A_1	∂A_2
k	$\lambda_{11}(k)$	$\lambda_{12}(k)$	$\lambda_{21}(k)$	$\lambda_{22}(k)$
1	0.01	0.54	0.79	0.3
2	0.19	0.78	0.71	0.31
3	0.43	0.38	0.35	0.91
4	0.77	0.11	0.33	0.75
5	0.62	0.13	0.58	0.46
6	0.85	0.13	0.37	0.76

Sieć FCM

Uczenie

$$e = \frac{1}{(t_e - 1) \cdot n} \cdot \sum_{t=1}^{t_e-1} \sum_{j=1}^n \epsilon_j,$$

$$\epsilon_j(t+1) = |\lambda'_j(t+1) - \lambda_j(t+1)|$$



Predykcja

$$\hat{\lambda}_{1i}(k+1) = f\left(\sum_{j=1}^c w_{ij} \cdot \lambda_{1j}\right),$$

$$\hat{\lambda}_{2i}(k+1) = f\left(\sum_{j=1}^c w_{ij} \cdot \lambda_{2j}\right),$$

Predykcja na poziomie pojęciowym (drugim poziomie granulacji)

$$j_{1k} = \arg \max_{j=1,2,\dots,c} (\lambda_{1j}(k))$$

$$j_{2k} = \arg \max_{j=1,2,\dots,c} (\lambda_{2j}(k))$$

$$\{A(k)\} = \{A_{j_{11}}, A_{j_{12}}, \dots, A_{j_{1N}}\}$$

$$\{\partial A(k)\} = \{\partial A_{j_{21}}, \partial A_{j_{22}}, \dots, \partial A_{j_{2N}}\}$$

$$G^2 e_1 = \frac{1}{K} \sum_{k=1}^K 1 | A_{j_{1k}} \neq \hat{A}_{j_{1k}},$$

$$G^2 e_2 = \frac{1}{K} \sum_{k=1}^K 1 | \partial A_{j_{2k}} \neq \partial \hat{A}_{j_{2k}}$$

Predykcja na pierwszym poziomie granulacji

$$\hat{a} = \frac{\sum_{j=1}^c \hat{\lambda}_{1j}(k+1) \cdot a_j}{\sum_{j=1}^c \hat{\lambda}_{1j}(k+1)},$$

$$\hat{m} = \frac{\sum_{j=1}^c \hat{\lambda}_{1j}(k+1) \cdot m_j}{\sum_{j=1}^c \hat{\lambda}_{1j}(k+1)},$$

$$\hat{b} = \frac{\sum_{j=1}^c \hat{\lambda}_{1j}(k+1) \cdot b_j}{\sum_{j=1}^c \hat{\lambda}_{1j}(k+1)},$$

$$\hat{a} = \frac{\sum_{j=1}^c \hat{\lambda}_{2j}(k+1) \cdot a_j}{\sum_{j=1}^c \hat{\lambda}_{2j}(k+1)},$$

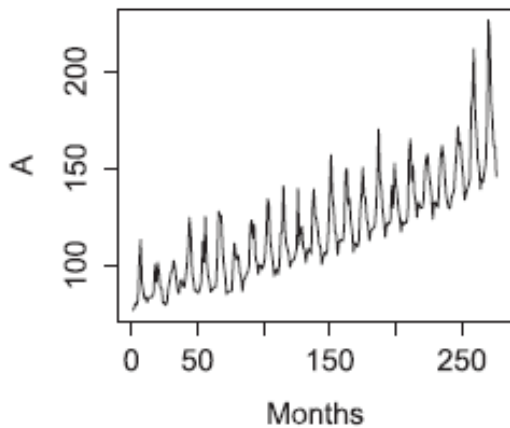
$$\hat{m} = \frac{\sum_{j=1}^c \hat{\lambda}_{2j}(k+1) \cdot m_j}{\sum_{j=1}^c \hat{\lambda}_{2j}(k+1)},$$

$$\hat{b} = \frac{\sum_{j=1}^c \hat{\lambda}_{2j}(k+1) \cdot b_j}{\sum_{j=1}^c \hat{\lambda}_{2j}(k+1)},$$

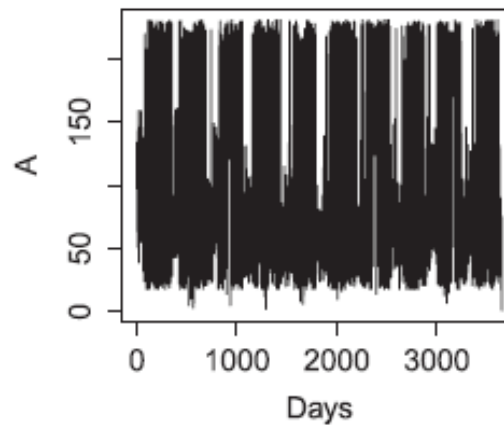
$$G^1 e_1^{Comp} = \frac{1}{K} \sum_{k=1}^K (1 - Comp(X_k, \hat{X}_k)),$$

$$G^1 e_2^{Comp} = \frac{1}{K} \sum_{k=1}^K (1 - Comp(\partial X_k, \partial \hat{X}_k)).$$

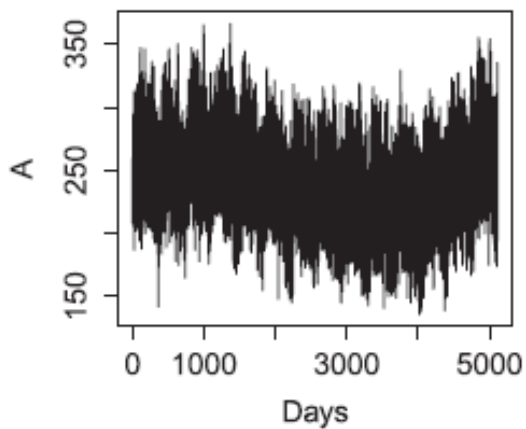
Eksperymenty



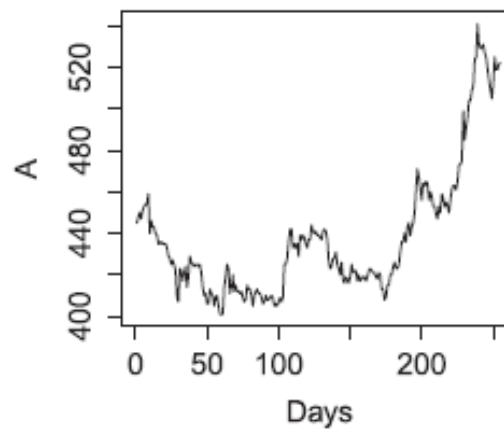
(a) Time series 2



(b) Time series 3



(c) Time series 4



(d) Time series 5

Błędy predykcji

Forecasting errors for time series 2.

	Number of clusters c				
	2	3	4	5	6
G^2E_1	0	0	0.64	0.64	0.65
$G^1E_1^{Match}$	0.47	0.30	0.30	0.32	0.34
G^2E_2	0.33	0.33	1	1	1
$G^1E_2^{Match}$	0.4	0.38	0.3	0.35	0.37

Forecasting errors for time series 3.

	Number of clusters c				
	2	3	4	5	6
G^2E_1	0.33	0.67	1	1	1
$G^1E_1^{Match}$	0.42	0.36	0.35	0.37	0.36
G^2E_2	0	1	1	1	1
$G^1E_2^{Match}$	0.20	0.17	0.14	0.14	0.14

Forecasting errors for time series 4.

	Number of clusters c				
	2	3	4	5	6
G^2E_1	0.75	0.5	0.5	0.75	1
$G^1E_1^{Match}$	0.46	0.44	0.47	0.48	0.49
G^2E_2	0.25	0.5	0.5	1	0.75
$G^1E_2^{Match}$	0.15	0.14	0.15	0.15	0.15

Forecasting errors for time series 5.

	Number of clusters c				
	2	3	4	5	6
G^2E_1	0.27	0.45	0.82	0.82	0.83
$G^1E_1^{Match}$	0.75	0.72	0.75	0.75	0.75
G^2E_2	0.09	0.44	0.65	0.82	0.91
$G^1E_2^{Match}$	0.61	0.59	0.57	0.58	0.58

Wniosek

Proponowana metoda umożliwia efektywne modelowanie i predykcję granularnych szeregów czasowych.

Dziękuję za uwagę